Structural Engineering Review, 1990, **2**, 141–159

REVIEW

Inelastic torsional behaviour of asymmetric buildings under severe earthquake shaking

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A comprehensive review is made of research studies on inelastic torsional coupling effects in asymmetric buildings subjected to severe earthquake ground motions. The aim has been to clarify the influence and relative importance of the key structural geometric and material parameters on dynamic inelastic torsional effects. These effects are measured in terms of the peak edge displacement and the peak element ductility demand in simplified asymmetric building models, which have been analysed parametrically in response to several strong motion earthquake records. Static loading analyses have also been carried out to investigate key issues in inelastic torsional response behaviour. Such analyses help explain the model dependency of the results of dynamic analyses of this problem, which have produced contradictory conclusions in past studies. Recommendations have been made for further research in order to gain a better understanding of inelastic torsional coupling, and hence to provide sound guidelines for the design of asymmetric buildings in active seismic zones.

1. INTRODUCTION

Buildings exhibit coupled torsional and translational responses to lateral ground motion input if their centres of floor mass and the centres of resistance do not coincide. Furthermore, torsional motions may occur even in nominally symmetric buildings because of two main reasons: namely, accidental eccentricity and torsional ground motions. The sources giving rise to accidental eccentricity include the difference between the assumed and actual distributions of mass and stiffness, asymmetric yielding strength and patterns of non-linear force–deformation relationships, and differences in coupling of the structural foundation with the supporting soil or rock. Torsional inputs arise from the wave propagation effects in horizontal ground motion and the rotational component of the ground motion about a vertical axis [1,2]. Therefore, the commonly used separate planar model approach [3], in which planar resisting structures (walls and/or frames) in the two orthogonal horizontal directions are analysed independently for the effects of the inplane horizontal component of ground motion, is very limited in application and may result in significant errors in some cases, even when the eccentricities of the centres of floor mass with respect to centres of resistance are small. Because the torsional motion creates additional internal forces and stresses in certain earthquake resistant elements of the structure, this effect must be taken into account by seismic building codes.

Systematic parametric studies have been carried out on the elastic torsional coupling effects in asymmetric building structures subject to earthquake ground motions, using either the idealized response spectrum approach [4–7] or a time history approach [8–10]. The results of these studies have been reviewed and the requirements for further research have been identified by Chandler in a previous article [1].

Unlike the studies of elastic torsional coupling effects in asymmetric buildings subjected to earthquake shaking, relatively little attention has been paid to the inelastic non-linear torsional and translational earthquake responses of such buildings. The general aims of aseismic design are to ensure that structures resist with slight or no damage a moderate intensity earthquake and to provide a large measure of resistance to prevent collapse or failure that might cause major property damage or loss of life when a severe earthquake occurs. Thus, in the first case, aseismic structures may be expected to remain within the elastic range or be excited slightly into the inelastic range. However, in the second case, it is unreasonably uneconomic to design structures to resist such an intensive earthquake elastically. Hence, aseismic structures may be excited well into the inelastic range during severe earthquake shaking. Therefore, conclusions drawn from studies of elastic behaviour of asymmetric buildings under earthquake ground excitation may not apply if yielding of resisting elements occurs. There is an important requirement to investigate the inelastic earthquake responses of asymmetric buildings by appropriate research to provide a basis for proposals to assess, and in some cases modify, current code torsional provisions in order to ensure reasonably conservative estimates of inelastic torsional effects.

A series of recent parametric studies have been carried out on the inelastic earthquake response of torsionally coupled buildings [11–19]. The major studies in this area differ significantly in terms of the variables defining the member layout and geometry, and the dynamic elasto-plastic behaviour of the structural models employed in the analysis. As a result of these differences, the various studies have drawn contradictory conclusions regarding the importance of torsional coupling in the post-yield earthquake response of asymmetric structures, and the sensitivity of key response parameters such as peak member ductility demand to the definition of the structural layout, dynamic structural properties and material behaviour. The objectives of this paper are to review the existing literature in this field and to clarify the reasons for the observed differences in the results and the conclusions drawn from dynamic response analysis. A series of static parametric analyses have been carried out which identify the key trends in the element ductility demand and ultimate lateral load resistance of characteristic structural models exhibiting torsional response behaviour. On this basis, a need is identified and proposals made for future research into the dynamic earthquake response of such structures based on appropriate understanding of previous work. In recommending such research, the need to employ realistic structural models based on simple and practical earthquake-resistant design procedures is emphasized.

2. SUMMARY OF EXISTING STUDIES

Kan and Chopra [11, 12] carried out the first parametric studies on the inelastic response of single-storey one-way eccentric structural models subjected to the 1940 El Centro earthquake record. The force–deformation relationship of the resisting elements was assumed to be elastic-perfectly plastic, and the yielding strength of the individual resisting elements was taken as proportional to their elastic stiffness. The multi-element model was simplified to a single element model (Fig. 1) by defining a circular yield surface in terms of the shear and torque acting on the system at and about the centre of resistance, respectively. The effects of torsional coupling were characterized by the lateral translation of the centre of mass, the rotation of the floor about the vertical axis through it, and the ratio of the total vector displacement of the corner columns to the displacement of the centre of mass (Fig. 2). The system parameters of this simple one-storey model were the eccentricity ratio e/r, where e is the static eccentricity between the centres of mass and resistance, and r is the mass radius of gyration about the centre of mass; the uncoupled torsional:translational frequency ratio $\Omega = \omega_\theta/\omega_x$; the uncoupled translational period T_x; and the damping

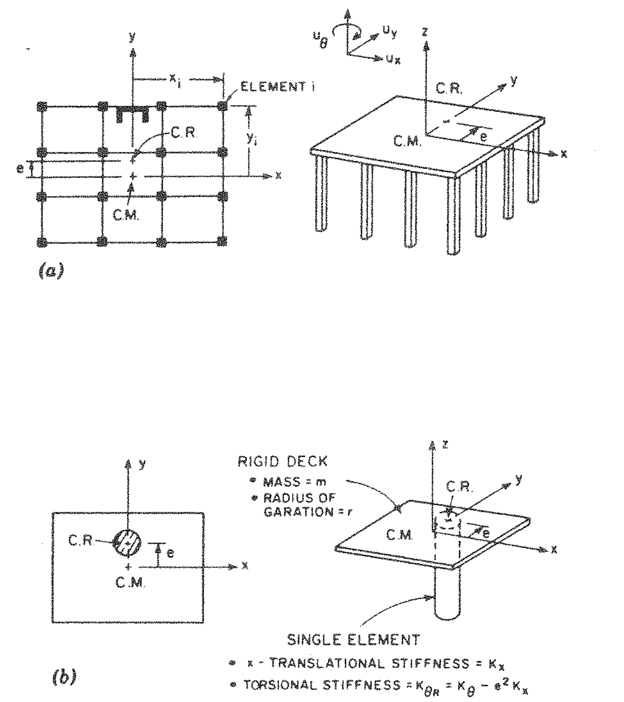

Fig. 1. One-storey structural model: (a) Idealized one-storey system; (b) its single-element model [12]. CM, centre of mass; CR, centre of resistance.

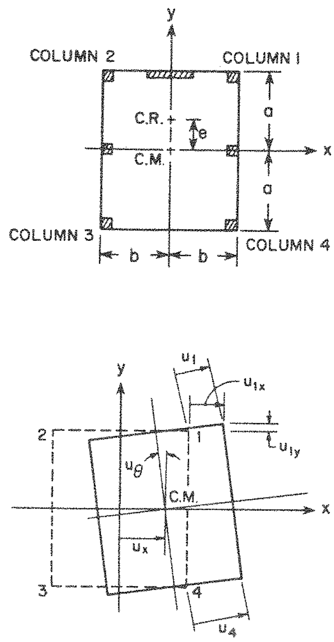

Fig. 2. Rectangular plan and its displaced configuration [12]. CM, centre of mass; CR, centre of resistance.

ratio. Kan and Chopra concluded that the effects of torsional coupling in the inelastic range depend significantly on the uncoupled torsional:translational frequency ratio, being most pronounced for systems with this ratio close to unity (Fig. 3). They found that for systems with an uncoupled frequency ratio larger than 2, the effect of torsional coupling on system and column deformations increases with increasing eccentricity ratio, but for systems with

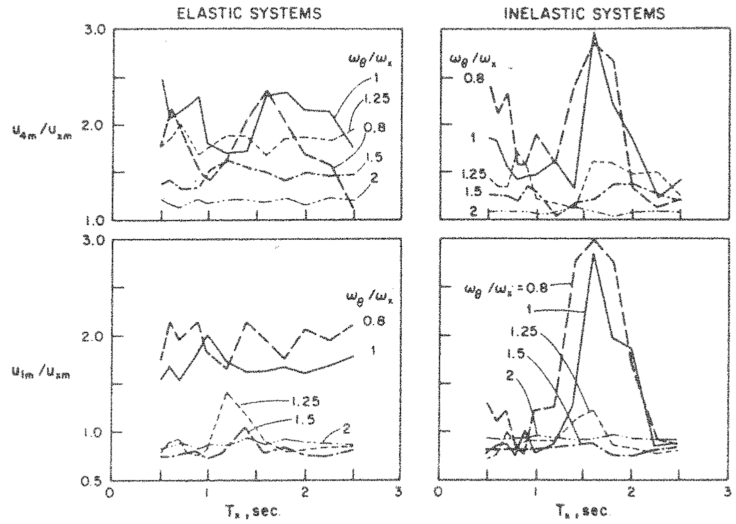

Fig. 3. Ratios of corner column deformation u_{im} to lateral displacement u_{xm} at CM for systems with $a/b = 1$ and $e/r = 0.4$ subjected to El Centro earthquake [12].

uncoupled frequency ratio smaller than 2, the effects of torsional coupling are complicated, giving no apparent systematic trends (Fig. 4). Finally, Kan and Chopra concluded that after the initial yielding, the system has a tendency to yield further, primarily in translation, and behave more and more like an inelastic single-degree-of-freedom system, responding primarily in translation.

Irvine and Kountouris [13] investigated the inelastic seismic response of a simple single-storey monosymmetric model having two identical resisting elements parallel to the direction of earthquake input. The eccentricity was caused by the offset of the centre of mass from the centre of stiffness. They found that the peak ductility demand of the worse affected element is insensitive to either the eccentricity ratio or uncoupled frequency ratio.

Syamal and Pekau [14] studied the inelastic response of a single-storey monosymmetric building model subjected to sinusoidal ground acceleration employing the Kryloff–Bogoliuboff method. Their single-storey model consisted of two elements parallel to the direction of the ground acceleration input and another two elements perpendicular to it (Fig. 5). The resisting elements were assumed to be bi-linear hysteretic. The eccentricity was caused by the unbalanced stiffness of the elements parallel to the direction of the ground acceleration. The yield displacements of these two elements were taken to be equal, and thus the yield strengths were proportional to their elastic stiffnesses. The system parameters characterizing the properties of the model were: the bi-linear coefficient of the resisting elements; the eccentricity ratio; the torsional:lateral frequency ratio of the corresponding torsionally uncoupled system, the amplitude of ground acceleration and the damping ratio. The response parameters were the peak ductility demand of the resisting elements and the response amplitudes of the translational and torsional displacements of the system. A parametric study was carried out to investigate the influences of the system parameters on response parameters. They found that, in contrast to the results of elastic parametric studies, the structure does not exhibit pronounced inelastic torsional coupling when the uncoupled torsional and lateral frequencies are close and the eccentricity is small (Fig. 6). Syamal and Pekau also found that the element peak ductility demand appears to be most pronounced for torsionally flexible structures, and the peak ductility demand of the element at the 'flexible' edge of the structure grows rapidly with increase in eccentricity (Fig. 7). For elements at the stiff edge, the ductility demand decreases only slowly with an increase in eccentricity ratio. Therefore, seismic building codes which reduce force requirements for these elements with increasing eccentricity ratio appear to substantially underestimate actual behaviour. Finally, Syamal and Pekau found that, although the uncoupled frequency ratio does not significantly affect the ductility demand of the element at the stiff edge, the ductility demand of the element at the flexible edge is critically affected by this parameter (Fig. 6).

Tso and Sadek [15] and Bozorgnia and Tso [16] carried out parametric studies to investigate the inelastic behaviour and the sensitivity of response parameters to system parameters of a simple single-storey one-way eccentric structural model subjected to earthquake ground excitation, using a step-by-step integration approach. The model used in their studies consists of a rigid rectangular floor deck of mass m supported by three resisting elements in the direction of the ground motion (Fig. 8). This model is statically indeterminate and the changes of initial eccentricity and the uncoupled torsional:translational frequency ratio were obtained by adjusting the elastic stiffness of the resisting elements and distances h of the elements from the centre of mass. The force–displacement relationship of the elements was assumed to be either bi-linear or bi-linear degrading. All elements were assumed to have the same yielding displacement. The peak element ductility demand and the displacement at the flexible edge were chosen as the response parameters. The system parameters were

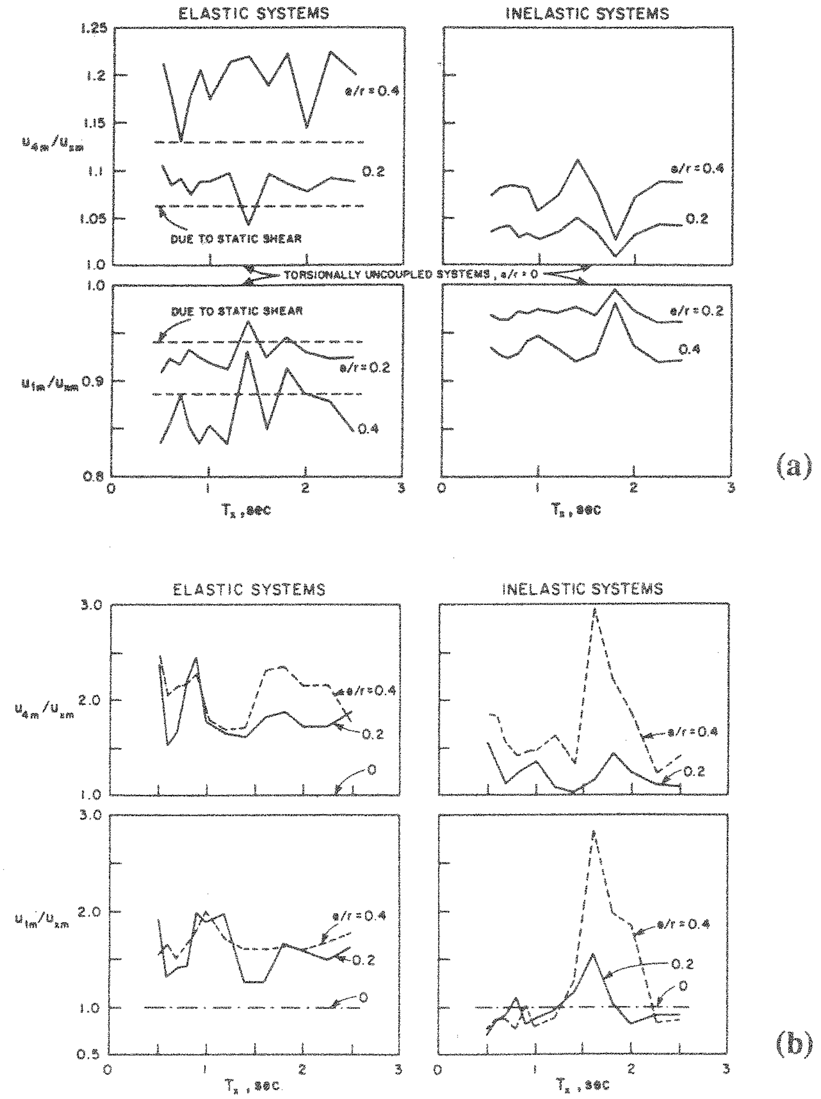

Fig. 4. Ratios of corner column deformation u_{im} to lateral displacement u_{xm} at CM for systems subjected to El Centro earthquake. (a) $\Omega = 2$ and $a/b = 1$; (b) $\Omega = 1$; $a/b = 1$ [12].

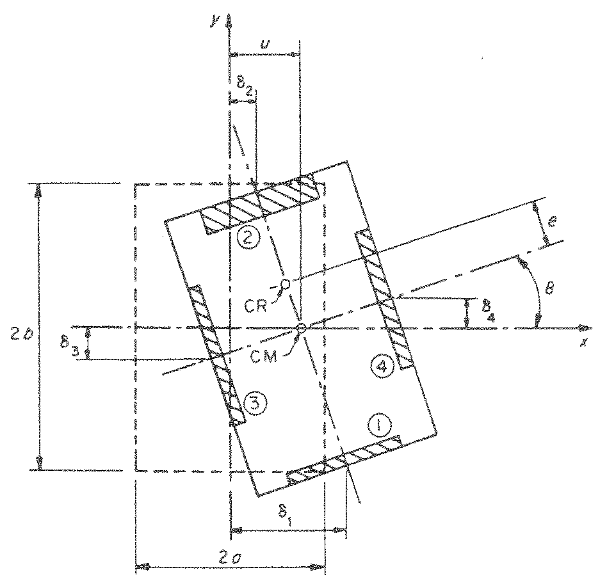

Fig. 5. Plan view of single-storey monosymmetric structural model [14].

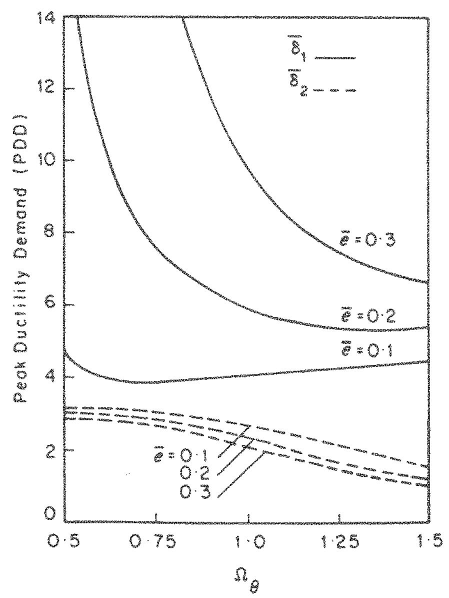

Fig. 6. Effect of uncoupled frequency ratio on peak ductility demand $\bar{\delta}$ of resisting elements 1 and 2 for varying eccentricity ratio $\bar{e} = e/r$ [14].

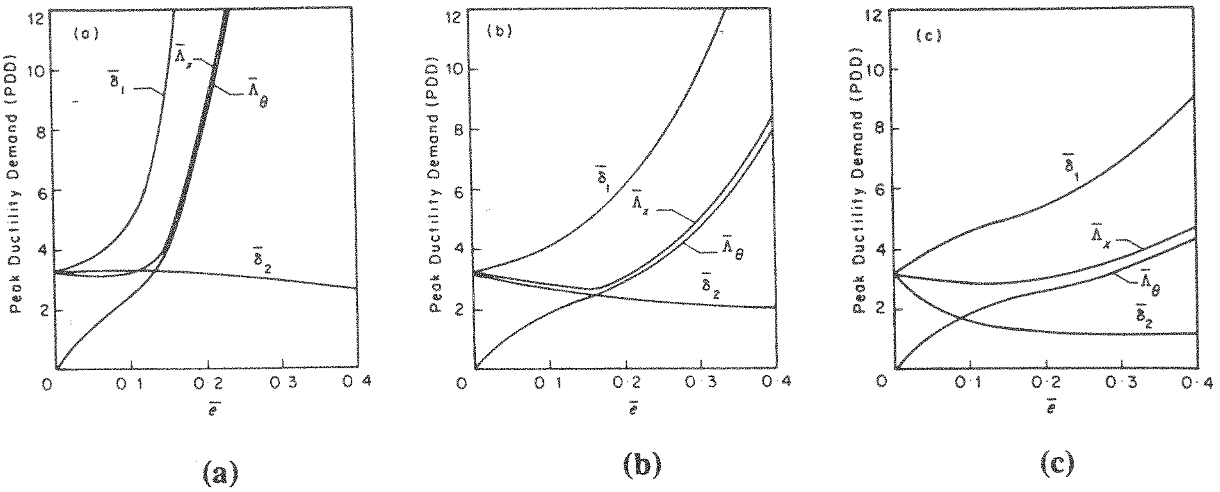

Fig. 7. Effect of eccentricity ratio $\bar{e}$ on peak ductility demand $\bar{\delta}$ of resisting elements 1 and 2, and translational and torsional system displacements $\bar{\Lambda}_x$, $\bar{\Lambda}_\theta$. (a) $\Omega = 0.5$; (b) $\Omega = 1.0$; (c) $\Omega = 1.5$ [14].

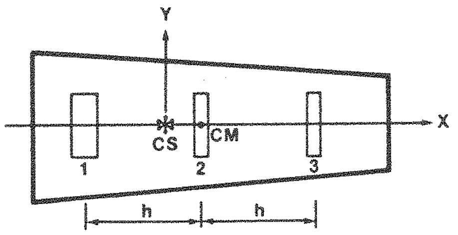

Fig. 8. Plan view of single-storey monosymmetric structural model [16]. CS, centre of stiffness; CM, centre of mass.

taken to be the initial eccentricity ratio, the uncoupled torsional : translational frequency ratio, the uncoupled translational period, and the excitation level parameter, which was the ratio of the elastic strength demand to the strength capacity of the system, assuming the system was symmetric. The results of extensive studies show that, firstly, unlike the results of elastic studies, the coincidence of uncoupled torsional and translational frequencies does not lead to abnormally high peak inelastic responses and that the element ductility demand is not sensitive to the uncoupled frequency ratio, with no systematic trends identified from the results (Fig. 9). Secondly, significant rotational motion is involved at the instant when the peak ductility demand is reached, which implies that the eccentric system does not respond primarily in translation when it is excited well into the inelastic range, as concluded by Kan and Chopra [12] (Fig. 10). Thirdly, these studies [15, 16] showed that eccentricity has a large effect on both element ductility demand (Fig. 11) and the flexible-edge displacement. Finally, it was concluded that the effect of asymmetry on the element ductility demand and on the flexible-edge displacement is most pronounced for stiff systems with low yield strength relative to the elastic strength demand (Figs. 9, 11).

In a later study, Tso and Bozorgnia [17] studied the maximum dynamic inelastic edge displacement and element deformation of a single-storey monosymmetric structural model subjected to unidirectional ground excitation. The concept of effective eccentricity, which was introduced for elastic one-way eccentric structures to evaluate the effect of asymmetry on the lateral displacement at the flexible edge of asymmetric buildings [18], was generalized for inelastic systems. The idea was to match the maximum dynamic displacement at the flexible edge of the asymmetric buildings to the edge displacement of the same building subjected to an equivalent static lateral load applied at a distance from the centre of stiffness equal to the effective eccentricity. In order to minimize the dependence of results on any individual record, six earthquake records were considered and the inelastic effective eccentricity was calculated by averaging the effective eccentricities obtained for these records. It was concluded that, except for short period structures having low yield strength relative to the elastic strength demand, the concept of effective eccentricity can be extended to responses of inelastic systems and the elastic effective eccentricity curves can provide a reasonable or conservative estimate of inelastic effective eccentricity. Hence these curves can be used to estimate the edge displacement and element deformation of inelastic systems.

Bruneau and Mahin [19] carried out parametric studies on the inelastic earthquake response of asymmetric single-storey systems with only two resisting elements in the direction of earthquake excitation. The post-yield characteristics of the resisting elements were assumed to be bi-linear hysteretic. Their results showed that the peak element ductility demand is not sensitive to any of the parameters; namely, the uncoupled frequency ratio, the eccentricity ratio and the uncoupled translational period, which have been considered to have significant influences on the response of such systems. Particularly, they found that element ductility demand increases with an increasing value of the uncoupled frequency ratio. Furthermore, they claimed by

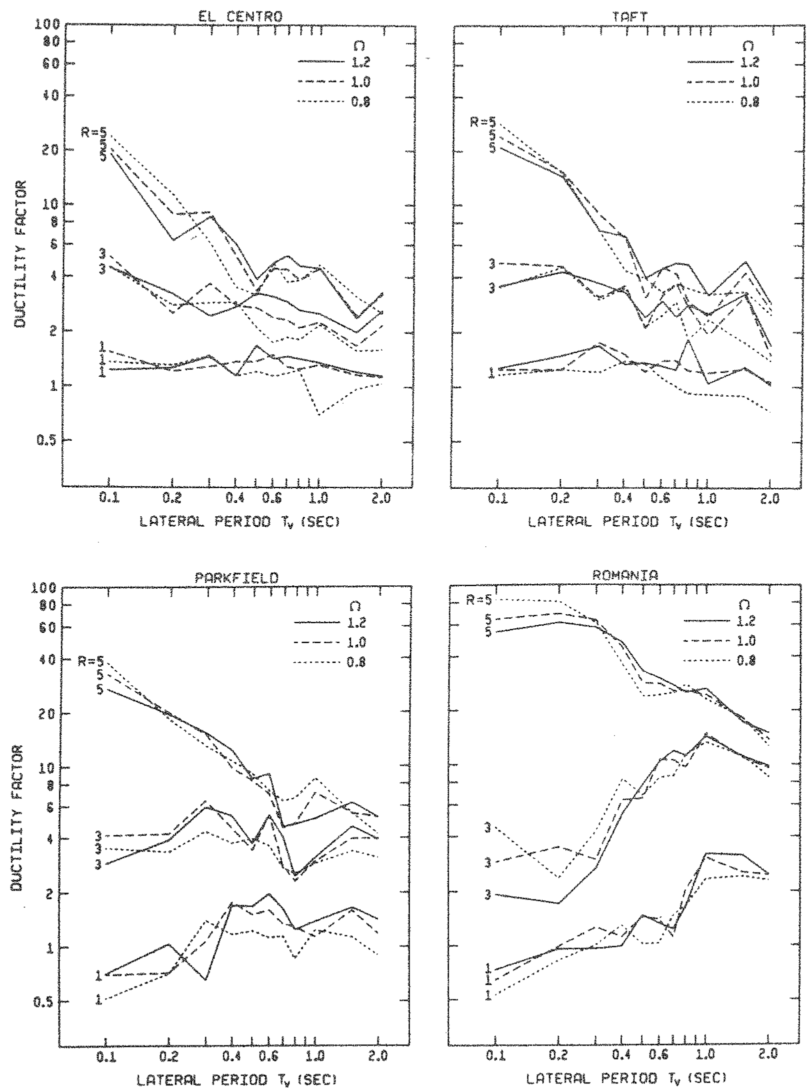

Fig. 9. Effect of uncoupled frequency ratio Ω on ductility demand of element 3. e/r = 0.25 [16].

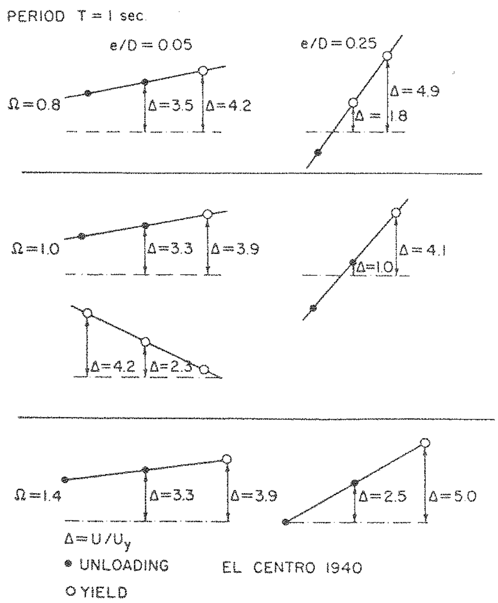

Fig. 10. Position of structure at instant of maximum edge displacement [15].

comparing results from the two element model and multi-element models that their two element model gives conservative estimates for the peak element ductility demand.

It is clear that in this research area investigators have employed different models and approaches to carry out their studies and that as a result contradictory conclusions have been obtained. The three main features of these different approaches and the conclusions drawn may be summarized as follows:

(1) Different structural models have been employed

Most researchers have used single-storey, mono-symmetric structural models to carry out their studies. The floor deck is considered rigid in its own plane with mass m supported by massless planar resisting elements, which are parallel to the direction of earthquake input and with eccentricity perpendicular to it. The structural elements are assumed inextensible and their torsional stiffness about their vertical axis is neglected.

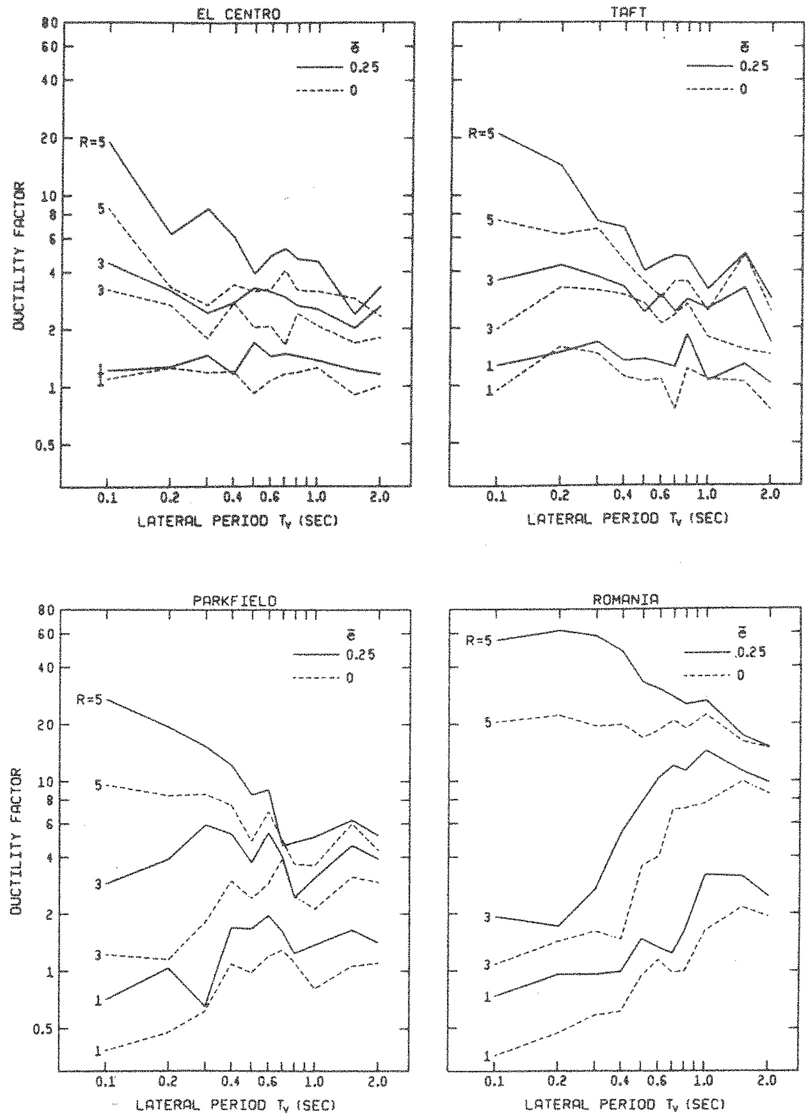

Fig. 11. Effect of eccentricity ratio $\bar{e}$ on the ductility demand of element 3. $\Omega = 1.2$ [16].

However, differences exist in the number of resisting elements used in the models. Kan and Chopra [11, 12] conducted their parametric studies based on a simplified model having only one resisting element with a circular base shear and torque yielding surface. Irvine and Kountouris [13], and Bruneau and Mahin [19] used a two element model; Tso and Sadek [15], and Tso and Bozorgnia [16, 17] used a three-element model; whereas Syamal and Pekau's model [14] has four elements, two parallel and two perpendicular to the direction of earthquake excitation.

Differences also exist in the definition and calculation of the values of some of the parameters. Irvine and Kountouris [13] considered eccentricity to be the result of eccentric mass, whilst the remaining researchers regarded eccentricity to be the result of unbalanced stiffness of the structural elements. In the definition of uncoupled torsional frequency, Kan and Chopra [12], Bruneau and Mahin [19], Syamal and Pekau [14], and Tso and Sadek [15] specified this parameter about the vertical axis through the centre of mass, whereas Irvine and Kountouris [13], and Tso and Bozorgnia [16, 17] specified it about the vertical axis through the centre of resistance. In order to get different values for the uncoupled torsional : translational frequency ratio, Bruneau and Mahin [19] adjusted the value of the radius of gyration of the floor slab; however, all other researchers achieved this by changing the distances of the resisting elements from the geometric centre of the floor slab.

Different response parameters were also used to characterize the torsional effects. Kan and Chopra [12] employed the ratio of the total vector deformation of the corner column to the translational displacement of the centre of mass (Fig. 2). Irvine and Kountouris [13], Bruneau and Mahin [19], and Syamal and Pekau [14] employed the peak ductility demand of the resisting elements. Tso *et al.* [15–17] employed both the peak ductility demand and the flexible-edge displacement.

(2) Contradictory conclusions have been drawn about the effect of uncoupled torsional to translational frequency ratio on the inelastic torsional response

In elastic studies of torsional coupling in asymmetric buildings subjected to earthquake excitations, all results show that structures with small eccentricity experience pronounced torsional coupling when the uncoupled torsional : translational frequency ratio (Ω) is close to unity. Nevertheless, in inelastic studies, different researchers reached contradictory conclusions about the effect of this parameter. Kan and Chopra [12] found that inelastic torsional coupling is significantly sensitive to Ω, the effect being most pronounced for systems with Ω close to unity. However, all other researchers found that unlike the elastic studies, the coincidence of uncoupled torsional to translational frequencies does not lead to pronounced high inelastic peak responses of torsionally coupled systems with low eccentricity subjected to earthquake excitations.

Contradictory conclusions have also been reached concerning the sensitivity and overall trends of element peak ductility demand to Ω. Bruneau and Mahin [19] found that the peak ductility demand is not very sensitive to Ω, but increases systematically with the increase of Ω. Results from the studies of Tso *et al.* [15, 16] show, however, that element ductility demand is not sensitive to the frequency ratio and no systematic trend of changes of element ductility ratio with changes in Ω can be seen from the results (Fig. 9). Syamal and Pekau [14] found that for sinusoidal ground excitation, although the ductility demand of the strong element is not very sensitive to Ω, the ductility demand of the weak element is critically affected by Ω, particularly for torsionally flexible systems having large eccentricity. Their results show that in all cases the ductility demand of the strong element falls slightly with the rise of Ω. For small eccentricities, the ductility demand of the weak element remains constant with changes of Ω. However, for moderate-to-large eccentricities, it drops rapidly with the increase of Ω (Fig. 6).

(3) Differing conclusions have been obtained about the effect of eccentricity on inelastic torsional coupling

In elastic studies, all results agree that as the eccentricity ratio (e/r) increases, the effects of torsional coupling on the earthquake forces increase; namely, the base shear and translational deformation decrease, whilst torque and torsional deformation increase. The conclusions from inelastic studies are more uncertain. Kan and Chopra [12] concluded that the effects of torsional coupling on system and column deformations depend on eccentricity in a complicated manner with no apparent systematic trends except for systems with $\Omega \geqslant 2$. In this case, the effects of torsional coupling increase with increasing eccentricity ratio (Fig. 4). Irvine and Kountouris [13], and Bruneau and Mahin [19] concluded that peak ductility demand is independent of the eccentricity ratio. These studies showed that differences in element ductility demand between eccentric and symmetric structures and between structures with different eccentricities remain small. On the other hand, Tso and Sadek [15], Tso and Bozorgnia [16], and Syamal and Pekau [14] found that ductility demand is very sensitive to eccentricity. An increase of over 100% in ductility demand was found to be not uncommon for systems with large eccentricity when compared with the response of systems with small eccentricity. Furthermore, the results showed that eccentricity has the effect of increasing the flexible-edge displacement of the structure by a factor of up to three [15], demonstrating that eccentricity is a critical parameter controlling the inelastic response of asymmetric structures to earthquake excitations.

3. DISCUSSION

3.1 Model dependency of inelastic earthquake response of asymmetric structures

The summary given in Section 2 shows that the interpretation of results from inelastic analysis of torsional coupling effects in asymmetric structures is a complicated issue. In the case of elastic analysis, studies using different models and approaches, and employing time history and response spectrum analysis, all reached similar conclusions about the effects of the different system parameters on the response of torsionally coupled structures. The situation is different in the case of inelastic analysis because, unlike elastic studies, the inelastic response is model dependent. Considering the single-storey, monosymmetric structural model shown in Fig. 12, if the earthquake acceleration input is unidirectional and parallel to the Y axis, then only two degrees of freedom are concerned, namely the translational displacement of the centre of resistance (CR), v, and the rotational movement of the floor slab about the vertical axis through CR, θ. Thus, in the elastic range, if damping is neglected, the equations of motion can be written as:

$$\begin{bmatrix} m & me_r \\ me_r & mr^2(1+e_r^{*2}) \end{bmatrix} \begin{Bmatrix} \ddot{v} \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} \Sigma k_i & 0 \\ 0 & \Sigma k_i x_i^2 \end{bmatrix} \begin{Bmatrix} v \\ \theta \end{Bmatrix} = -\begin{Bmatrix} m\ddot{v}_g \\ me_r\ddot{v}_g \end{Bmatrix} \quad (1)$$

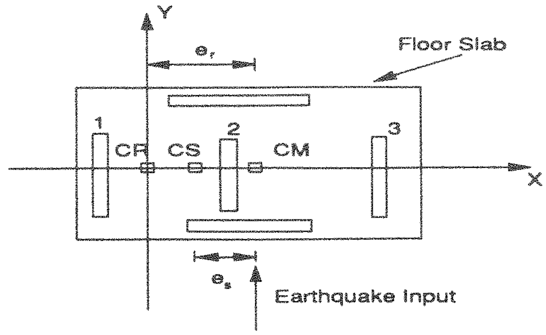

Fig. 12. Plan view of single storey monosymmetric structural model.

in which m is the mass of the floor deck, r is the radius of gyration of the deck about the vertical axis through the centre of mass (CM), $e_r{}^*$ is the stiffness eccentricity ratio e_r/r, and k_i is the translational stiffness of element i parallel to the direction of the earthquake input.

Equation (1) can be rewritten as follows:

$$\begin{bmatrix} 1 & e_r^* \\ e_r^* & 1+e_r^{*2} \end{bmatrix} \begin{Bmatrix} \ddot{v} \\ r\ddot{\theta} \end{Bmatrix} + \omega_y^2 \begin{bmatrix} 1 & 0 \\ 0 & \Omega^2(1+e_r^{*2}) \end{bmatrix} \begin{Bmatrix} v \\ r\theta \end{Bmatrix} = - \begin{Bmatrix} \ddot{v}_g \\ e_r^* \ddot{v}_g \end{Bmatrix} \quad (2)$$

in which $\Omega = \omega_\theta/\omega_y$ is the uncoupled torsional:translational frequency ratio. Therefore, it is apparent that the elastic responses v and $r\theta$ are uniquely determined by the system's parameters, $e_r{}^*$, Ω, ω_y, damping and the earthquake input. Systems having the same values of $e_r{}^*$, Ω, ω_y and damping, and subjected to the same earthquake excitation will have the same elastic responses, v and $r\theta$.

For the analysis of inelastic response, the equations of motion can be written in incremental form:

$$[m]\{\Delta\ddot{v}\} + [K(t)]\,\{\Delta v\} = - \begin{Bmatrix} \Delta\ddot{v}_g \\ e_r \Delta\ddot{v}_g \end{Bmatrix} \quad (3)$$

in which

$$[m] = \begin{bmatrix} 1 & e_r^* \\ e_r^* & 1+e_r^{*2} \end{bmatrix} \quad (4)$$

$$\{\Delta\ddot{v}\} = \begin{Bmatrix} \Delta\ddot{v} \\ r\Delta\ddot{\theta} \end{Bmatrix} \quad (5)$$

$$[K(t)] = \begin{bmatrix} \dfrac{\Sigma k_i(t)}{m} & \dfrac{\Sigma k_i(t) x_i}{mr} \\ \dfrac{\Sigma k_i(t) x_i}{mr} & \dfrac{\Sigma k_i(t) x_i^2}{mr^2} \end{bmatrix} = \omega_y^2(t) \begin{bmatrix} 1 & s^*(t) \\ s^*(t) & \Omega^2(t)\,(1+e_r^{*2}) \end{bmatrix} \quad (6)$$

$\omega_y\,(t)$ and $\Omega(t)$ may be considered as the instantaneous translational circular frequency and torsional:translational frequency ratio at time t; $s(t)$ is the instantaneous position of the centre of resistance at time t; $s^*(t) = s(t)/r$.

From Equation (3), it is clear that the inelastic response not only depends on the system's parameters, $e_r{}^*$, Ω and ω_y but also on the number, location, force–deformation relationship and yield strength of the individual resisting elements. Therefore, the inelastic response is highly model dependent. The contradictory conclusions summarized in Section 2 are the direct result of the different analytical models used by the various investigators.

Since the inelastic response is model dependent, and given that the overall objective at the present stage of research is to obtain a better understanding of the inelastic behaviour of asymmetric structures under severe earthquake excitations and to clarify the influence of the system parameters on inelastic torsional effects, an independent assessment of the analytical models employed by different researchers and the results obtained has the highest priority for further research work. The suitability of an analytical model should be decided after consideration of the following factors:

(1) reliability for ensuring conservative estimates of the effect of inelastic torsional coupling on structural response parameters;

(2) realistic representation of a range of actual building structures, and ease of interpretation of the results, and

(3) simplicity of model definition and subsequent analysis.

Kan and Chopra's single-element model [11,12] is simple and enables straightforward parametric studies to be carried out. However, although it can predict the global translational and torsional responses of the system, it is too simple to provide a direct insight into the ductility demand of the individual resisting elements. This makes the application of the single-element model very limited and unsuitable for the analysis of inelastic torsional effects.

The two-element model employed by Irvine and Kountouris [13] and Bruneau and Mahin [19] can predict both the global response of the system and the ductility demand of the structural elements, whilst at the same time retaining simplicity. However, the two-element model is statically determinate. In Section *4* of this review, the statically determinate two-element model is demonstrated to underestimate the peak ductility demand of the weak element. Therefore, the two-element model is not the most suitable for inelastic analysis of torsional effects.

The three-element model employed by Tso *et al.* [15–17] seems to be the most suitable overall. This model is statically indeterminate, the structural form encouraged in aseismic design [20]; therefore it is a more realistic model. The expected conservatism of this model should be studied in further research work by comparing the results with some of those obtained from models having more than three elements.

3.2 Definition and influence of the uncoupled torsional to translational frequency ratio

There are two alternative methods for varying the uncoupled torsional:translational frequency ratio Ω whilst keeping the other system parameters constant. The first is to change the radius of gyration of the floor deck r, and the second is to adjust the distances of the resisting elements from the geometric centre of the floor plan. In their study, Bruneau and Mahin [19] adopted the first approach. Changing the value of r is equivalent to changing the aspect ratio of the building. This is rarely possible in practice because of constraining architectural requirements. In order to vary Ω, a practical engineering solution is to adjust the distribution of the stiffness rather than the aspect ratio of the building. Furthermore reducing the value of r whilst fixing other parameters (particularly the system's torsional stiffness) in order to increase Ω decreases the rotational inertia mr^2 of the floor slab and therefore reduces the system's capacity for torsional resistance. This gives rise to greater rotational movement. Because Bruneau and Mahin [19] fixed the distances of the resisting elements to the geometric centre of the floor deck, a higher torsional response leads to higher element deformation. Hence, in their study the weak element ductility ratio always increases with increasing value of Ω. This differs from the findings of Tso *et al.* [15, 16], and Syamal and Pekau [14] who adopted the second alternative, in which a higher value of Ω corresponds to a higher torsional stiffness. This is achieved by increasing the distances of the resisting elements to the geometric centre of the floor plan. In this approach, increasing the value of Ω has two contradictory effects on element ductility demand. On the one hand, higher torsional stiffness makes the structure torsionally stiffer, therefore reducing the system's torsional response. On the other hand, the effect of rotational motion on element deformation increases with increasing distance of the element to the geometric centre of the floor plan. Consequently, the effect of Ω on element ductility demand is complicated. The studies conducted by Tso and Bozorgnia [16], and Syamal and Pekau [14] disagreed on the effect of Ω on the peak ductility demand of the weak element. The former concluded that Ω is a less critical parameter for the element ductility demand, whilst the latter found that although the strong element is not influenced significantly by this parameter, the weak element ductility demand is affected critically by Ω, particularly for torsionally flexible systems with large eccentricity. The main reason why the above studies reached different conclusions about the effect of Ω on element ductility demand appears to be that the range of the parameters, $\Omega = 0.8 - 1.2$, $e_r{}^* = 0.25$, chosen by Tso and Bozorgnia [16] for their parametric study is not wide enough to reveal the real trends. In particular, they omitted the range of parameters representing the characteristics of torsionally flexible structures having high eccentricities ($\Omega < 0.8$ and $e_r{}^* > 0.25$). Further research on this topic is therefore needed before final conclusions are drawn. The direct comparison of results obtained in references [14] and [16] is made more difficult by the different reference points chosen to define the rotation movement, hence leading to different definitions of Ω. In deriving the equations of motion for single-storey, monosymmetric structural models, there are two different approaches to define the rotational movement of the floor deck. The first is to define it about the centre of mass (CM); the alternative is to define it about the CR. These two approaches lead to different definitions of the uncoupled torsional:translational frequency ratio, written as Ω_m if the reference point is CM and Ω_r if the reference point is CR. In addition, Rutenberg and Pekau [10] suggested a third definition as follows:

$$\Omega_0^2 = \frac{\omega_{\theta_0}^2}{\omega_y^2} = \frac{K_{\theta_r} m}{J_{\theta_m} K_y} = \frac{K_{\theta_r}}{K_y r^2} \tag{7}$$

in which K_{θ_r} is the system's torsional stiffness about CR and J_{θ_m} is the mass moment of inertia of the floor slab about CM.

The relationships between Ω_m, Ω_r and Ω_0 can be derived using the following two equations:

$$J_{\theta r} = J_{\theta m} + me_r{}^2 \tag{8}$$

$$K_{\theta m} = K_{\theta r} + K_y e_r{}^2 \tag{9}$$

Thus

$$\Omega_m^2 = \frac{\omega_{\theta m}^2}{\omega_y^2} = \frac{K_{\theta m} m}{J_{\theta m} K_y} = \frac{(K_{\theta r} + K_y e_r{}^2)}{K_y r^2} = \Omega_0^2 + e_r{}^{*2} \tag{10}$$

Similarly

$$\Omega_r^2 = \frac{\Omega_0^2}{(1 + e_r{}^{*2})} \tag{11}$$

Therefore, $\Omega_r^2 \leqslant \Omega_0^2 \leqslant \Omega_m^2$ and the relationship between Ω_m and Ω_r is given by:

$$\Omega_m = [\Omega_r^2(1 + e_r{}^{*2}) + e_r{}^{*2}]^{1/2} \tag{12}$$

$$\Omega_r = \left[\frac{\Omega_m^2 - e_r^{*2}}{1 + e_r^{*2}}\right]^{1/2} \tag{13}$$

The relationship between Ω_m and Ω_r is plotted in Fig. 13 for selected values of $e_r{}^*$.

Tso and Bozorgnia's [16] models with the parameters $e_r{}^* = 0.25$ and $\Omega_r = 0.8 - 1.2$ have the corresponding $\Omega_m = 0.862 - 1.262$. Results from Syamal and Pekau [14] show that for models having these parametric values, element ductility demand is not sensitive to Ω_m, but for smaller values of Ω_m or larger values of $e_r{}^*$, the weak element ductility demand drops rapidly with increasing value of Ω_m (Fig. 6).

A detailed discussion on the advantages and disadvantages of chosing CM or CR as the reference point has been given by Bruneau and Mahin [19]. Until now, the choice of the reference centre to define the rotational motion seems to have been a matter of researcher's preference. However, as indicated in reference [19], Chapter 2.1.3, if CM is chosen as the reference centre

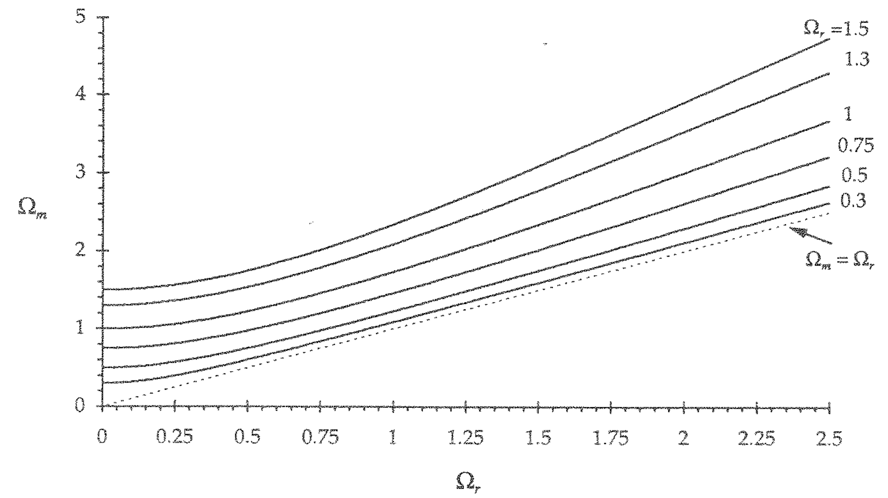

Fig. 13. Relationship between Ω_m and Ω_r.

and the uncoupled frequency ratio is defined as Ω_m, some combinations of eccentricity and uncoupled frequency ratio (relating to low uncoupled frequency ratio Ω_m and large eccentricity ratio e_r*) have no physical meaning; that is, no real physical systems can be represented since the system's first eigenpair does not exist. If CR is chosen as the reference centre and the uncoupled frequency ratio is defined as Ω_r, all combinations of system parameters are possible, having corresponding physical systems. Thus, for parametric studies of single-storey models CR may be a better choice than CM, bearing in mind that in this case the centre of resistance is easily defined. However, for multistorey buildings, especially irregular structures, the locations of the centres of resistance are load dependent and may vary significantly from floor to floor [21].

3.3 Stiffness and strength eccentricities

The contradictory conclusions about the sensitivity of element ductility demand to the system's eccentricity ratio may also be considered to be the result of the different model definitions employed by the various researchers. Studies on two element models [13, 19] have shown that element ductility demand is independent of eccentricity, whilst studies on three-element models [15, 16] show the opposite effect. Since most building structures have more than two resisting elements, the two-element model seems to be oversimplified and underestimates the effect of eccentricity on element ductility demand, hence giving non-conservative results. The lack of dependence of element ductility demand on eccentricity shown by dynamic analysis of two-element models may be understood by considering the behaviour of this model in static analysis (see Section *4*). Because the two-element model is statically determinate, the element forces are independent of the distribution of element stiffness which is characterized by the model's eccentricity. However, in static analysis of the three-element model the element forces are dependent on the distribution of element stiffness, and the ductility demand of the weak element is highly sensitive to eccentricity for systems having small-to-moderate values of uncoupled frequency ratio. On this point, the results of dynamic and static analysis are in full agreement.

In elastic analysis, only one eccentricity parameter, the stiffness eccentricity ratio e_r*, needs to be considered. But in inelastic analysis, asymmetric element yielding strength and varying non-linear force-deformation relationships also give rise to torsional coupling. Because yielding changes the stiffness of the resisting elements and moves the centre of resistance away from the yielding element, systems with initially symmetric stiffness but eccentric yielding strength will exhibit torsional coupling as soon as the first yielding occurs. There are three sources which lead to an asymmetric distribution of system's strength, namely different types of structural system (frames and shear walls, for instance) employed to resist the lateral loads, uncertainties in calculating the yielding strength of the structural elements, and statistical variations of the strength of the materials used. The strength eccentricity is expected to have a major effect on the inelastic response of asymmetric buildings to earthquake excitation. Therefore, in inelastic studies, the system's strength eccentricity must be taken into account even for systems having an initially symmetric distribution of stiffness.

Irvine and Kountouris [13] studied a two-element model with symmetric stiffness and strength but with eccentric distribution of mass. All other studies [12, 14–17, 19] considered the strength eccentricity to be equal to the stiffness eccentricity by assuming that all elements have the same yielding displacement, as in Fig. 23. Thus, in the models employed in these studies an element's yielding strength is proportional to its stiffness and the centre of strength (CS) coincides with the CR. Until now, little attention has been paid to the effect of strength eccentricity on inelastic torsional coupling. Bruneau and Mahin [19] studied some very simple cases of the inelastic response of initially symmetric two-element models having unequal yielding levels $0.8F_y$ and F_y, F_y and $1.2F_y$, F_y and $1.5F_y$, F_y and $2.0F_y$, without taking the strength eccentricity as a separate parameter. Sadek and Tso [22] introduced the strength eccentricity concept and proposed to use it as a measure of the degree of asymmetry. They carried out parametric studies based on a single-storey, monosymmetric structure having four columns, the model being equivalent to a two-element model. The conclusion of this study that stiffness eccentricity is not influential in determining the element peak ductility demand is contradictory to that of their previous studies using three-element models. Further parametric studies need to be carried out to investigate the influence of strength eccentricity on inelastic torsional coupling based on three-element models, and in order to demonstrate its effect the strength eccentricity ratio should be defined as an independent system parameter.

4. STATIC PARAMETRIC ANALYSIS OF TORSIONAL COUPLING EFFECTS IN ASYMMETRIC STRUCTURES

4.1 Introduction

Previous studies have concentrated mainly on analysis of dynamic time history responses. Few researchers have compared the results from different structural models. Tso and Sadek [15] compared the ductility demand of the element furthest away from the centre of resistance employing their three-element model with the corresponding results from Irvine and Kountouris's two-element model [13]. They found that, in general, Irvine's two-element model results in lower ductility demand than the three-element model (Fig. 14). Thus,

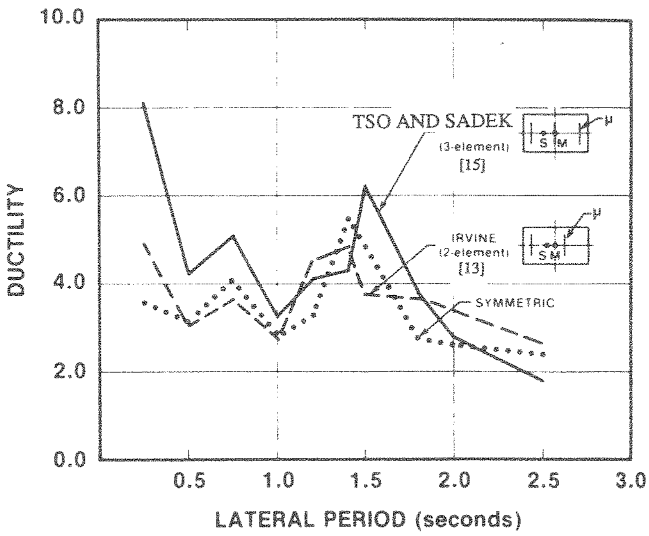

Fig. 14. Comparison between ductility demands μ in 3-element, 2-element and symmetric models subjected to El Centro earthquake [15]. S, centre of stiffness; M, centre of mass.

Tso and Sadek's conclusion is that the two-element model tends to underestimate element ductility demand. However, the results of Bruneau and Mahin's study [19] showed the opposite trend. In order to demonstrate their two-element model to be appropriate and conservative, they compared the ductility ratio of the weak-edge element obtained from models having two, four, six and eight elements, respectively. They found that the ductility ratios of the weak-edge element obtained from a two-element model are conservative and that the differences are not significant when compared with results obtained from multi-element models. Furthermore, for all models this ductility ratio was found to be greater with an increasing value of the uncoupled frequency ratio. The latter trend has been explained in Section 3.2. The former conclusion is invalidated by the fact that the various models employed in Bruneau and Mahin's study [19] have different values of the key parameters; that is, besides the different numbers of resisting elements, the models have unequal values of the total yielding base shear, with models with more resisting elements having higher yielding base shear than the two-element model. The models also have different strength eccentricities. Bruneau and Mahin scaled the intensity of the earthquake records in order to match the maximum elastic displacement of an equivalent SDOF system to that of the two or multi-element eccentric system's weak-edge element. Thus, the two-element system and the multi-element systems not only have different yield base shear and intensities of earthquake input, which may cause the system parameter reflecting the ratio of the intensity of the earthquake input to the structure's yield base shear to vary, but they also have different strength eccentricities. For the purpose of comparing results from models having different numbers of elements, it is essential for the models to have the same values for all the system parameters, leaving the number of resisting elements to be the only difference. This is not the case in Bruneau and Mahin's study [19].

In order to gain a better understanding of the model dependency of inelastic torsional effects in asymmetric structures and to choose an analytical model which ensures conservative estimates of torsional effects and retains simplicity, it is helpful to carry out a static analysis to clarify the influences of the various system parameters on the response of different models. Section 4.4 gives some results from inelastic static parametric analysis employing models with two and three elements. Dynamic analysis of inelastic torsional response of such models will be the subject of further research.

4.2 Characteristics of the models employed

The models employed in the static analysis are single-storey, monosymmetric, having two or three resisting elements parallel to the direction of the applied loading as shown in Fig. 15. The floor slab is assumed to be rigid in its own plane and the mass is uniformly distributed on the floor. The resisting elements are considered massless

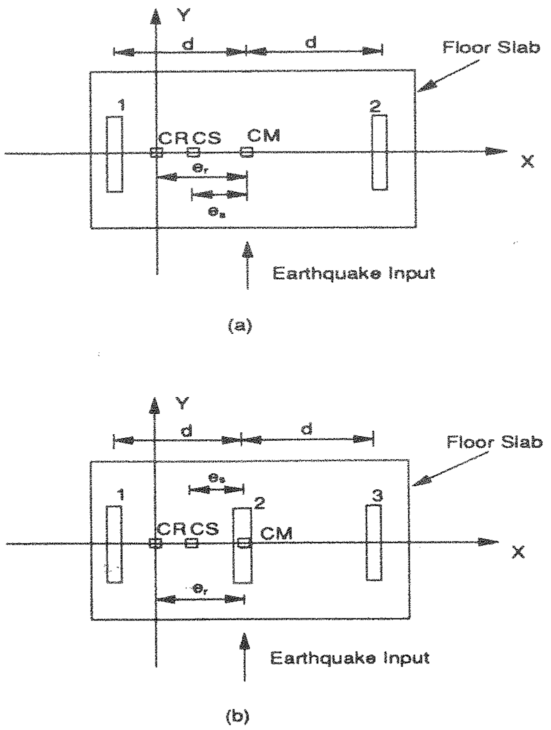

Fig. 15. Plan view of single-storey monosymmetric structural models employed in static parametric analysis: (a) 2-element model; (b) 3-element model. CM, centre of mass; CR, centre of resistance; CS, centre of strength.

and inextensible. Their torsional stiffness about their own vertical axis and translational stiffness perpendicular to their own acting plane are neglected. To simplify the analysis, the force–deformation (end shear and end lateral displacement) relationship of the elements is assumed to be elastic-perfectly plastic.

There are three reference centres associated with each model, namely the CM, CR and CS characterizing the distribution of mass, stiffness and strength of the model respectively. CM coincides with the geometric centre of the floor plan in all cases. The traditional definition of CR is employed, giving:

$$x_r = \frac{\Sigma k_i x_i}{\Sigma k_i} = 0 \quad (14)$$

$$y_r = 0 \quad (17)$$

and the stiffness eccentricity e_r is defined as the offset of CR from CM. The stiffness eccentricity ratio $e_r{}^* = e_r/r$, where r is the radius of gyration of the floor deck about CM, which is to be assumed constant.

The CS is defined as the point through which the resultant of all the element forces acts when all elements are loaded to their yield strengths. The coordinates of CS can be found by taking the first moment of the yield strengths about the origin of coordinates as follows:

$$x_s = \frac{\Sigma F_{yi} x_i}{\Sigma F_{yi}} \quad (16)$$

$$y_s = 0 \quad (17)$$

and the strength eccentricity e_s is defined as the offset of CS from CM. The strength eccentricity ratio $e_s{}^* = e_s/r$.

The system's total yield base shear $F_y = \Sigma F_{yi}$, where F_{yi} is the yield strength of the *ith* element. The system's total translational stiffness $K_y = \Sigma k_i$, where k_i is the translational stiffness of the *ith* element in the y direction. The system's torsional stiffness about CR, $K_{\theta r} = \Sigma k_i x_i{}^2$. Eccentricities are caused by the unbalanced stiffnesses and strengths of the two edge elements. Changes in stiffness eccentricity and torsional stiffness $K_{\theta r}$ are achieved by adjusting the difference in stiffness and the distances to CM (d) of the two edge elements. For example, a higher value of $K_{\theta r}$ combined with a constant value of $e_r{}^*$ can be achieved by increasing the distance d and reducing the difference in translational stiffness of the two edge elements. In this paper, changes in Ω_r are obtained by adjusting the value of torsional stiffness as opposed to changing r.

4.3 Determination of element properties from system parameters

4.3.1 Two element model

For the two element model, k_i, F_{yi} and d are uniquely determined by the values of the system parameters, as follows:

$$F_{y1} = \frac{1}{2} F_y \left(1 + \frac{e_s^* r}{d}\right) \quad (18)$$

$$F_{y2} = \frac{1}{2} F_y \left(1 - \frac{e_s^* r}{d}\right) \quad (19)$$

$$k_1 = \frac{1}{2} K_y \left(1 + \frac{e_r^* r}{d}\right) \quad (20)$$

$$k_2 = \frac{1}{2} K_y \left(1 - \frac{e_r^* r}{d}\right) \quad (21)$$

$$d = r\,(\Omega_r^2(1 + e_r^{*2}) + e_r^{*2})^{1/2} \quad (22)$$

4.3.2 Three element model

Unlike the two element model, k_i, F_{yi} and d of the three-element model cannot be determined uniquely by the system parameters. For a given set of system parameters, many combinations of k_i, F_{yi} and d are possible. In this paper, the stiffness and yielding strength of element 2 are taken to be the average values of all three elements. Thus:

$$F_{y1} = F_y \left(\frac{1}{3} + \frac{1}{2}\frac{e_s^* r}{d}\right) \quad (23)$$

$$F_{y2} = \frac{1}{3} F_y \quad (24)$$

$$F_{y3} = F_y \left(\frac{1}{3} - \frac{1}{2}\frac{e_s^* r}{d}\right) \quad (25)$$

$$k_1 = K_y \left(\frac{1}{3} + \frac{1}{2}\frac{e_r^* r}{d}\right) \quad (26)$$

$$k_2 = \frac{1}{3} K_y \quad (27)$$

$$k_3 = K_y\left(\frac{1}{3} - \frac{1}{2}\frac{e_r^* r}{d}\right) \quad (28)$$

$$d = r\left(\frac{3}{2}(\Omega_r^2\,(1 + e_r^{*2}) + e_r^{*2})\right)^{1/2} \quad (29)$$

4.4 Results of static analysis

A static concentrated load F is applied to each model through CM and parallel to the y axis. Closed form solutions for the ultimate value of load F (denoted as F_{max}) which the model can withstand and the maximum ductility demand of the resisting elements have been derived as given below.

4.4.1 Results from two-element models

The two-element model is a statically determinate system. Because of this, the distribution of the load F between the two elements is independent of the distribution of stiffness. Moreover, as soon as the weak element yields, the system becomes a mechanism assuming the post-yielding force–deformation relationship of the elements is elastic-perfectly plastic. There is therefore no development of plasticity in the material of the elements. The maximum ductility ratio of the elements is 1.0, irrespective of e_r^* and e_s^*. After first yielding, the problem becomes dynamic and is therefore beyond the scope of static analysis.

The maximum value of lateral load F is:

$$F_{\max} = F_y\left(1 - \frac{e_s^* r}{d}\right)$$

$$= F_y\left(1 - \frac{e_s^*}{(\Omega_r^2(1 + e_r^{*2}) + e_r^{*2})^{1/2}}\right) \quad (30)$$

To demonstrate the effect of e_s^* and Ω_r on F_{max}, equation (30) has been plotted in Figs 16 and 17. In Fig. 16, it is assumed that the element yield strengths are proportional to their stiffness (see Fig. 23), therefore $e_s^* = e_r^*$. In Fig. 17, e_r^* is taken to be zero, representing initially symmetric systems (see Fig. 18). From Figs 16 and 17, it is clear that in static analysis both strength eccentricity and uncoupled frequency ratio significantly affect the ultimate load the structure can withstand. The effect of strength eccentricity is particularly pronounced for torsionally flexible structures ($\Omega_r = 0.5 - 0.8$) with small-to-moderate strength eccentricities ($e_s^* = 0.0 - 0.4$); in these cases, F_{max} drops rapidly with increasing e_s^*. In all cases, F_{max} increases with increasing value of Ω_r. The

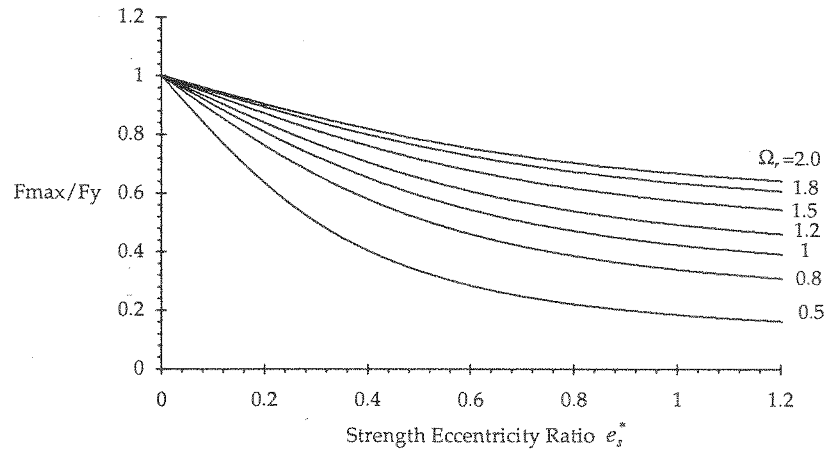

Fig. 16. Effect of strength eccentricity ratio on F_{max} for 2-element models with equal strength and stiffness eccentricity.

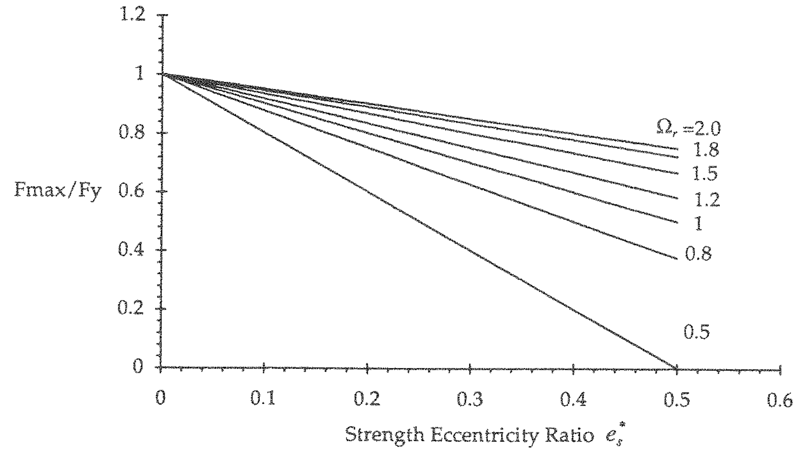

Fig. 17. Effect of strength eccentricity ratio on F_{max} for 2-element models with evenly distributed element stiffness.

effect of Ω_r on F_{max} is more critical for structures having moderate-to-large strength eccentricities ($e_s^* \geq 0.4$).

4.4.2 Results from three-element models

The three-element model is statically indeterminate. Following first yielding, the system becomes statically determinate and can still sustain an increasing load F. When a second element yields, the system becomes a mechanism and the static load reaches its ultimate value F_{max}. At this stage, there has been some development of material plasticity in the element which yields first, and hence its ductility demand is larger than 1.0. Therefore, in static analysis of the effect of torsional coupling on the ductility demand of resisting elements, the two-element model underestimates the ductility demand compared with the three-element model, since in the former case no development of plasticity is possible.

4.4.2.1 Initially symmetric systems

Initially symmetric systems are chosen to demonstrate the effect of strength eccentricity and remove the effect of stiffness eccentricity. For initially symmetric systems, all three elements have the same elastic stiffness but unbalanced yielding strength as shown in Fig. 18. Therefore, CR coincides with CM but CS does not; that is, $e_r^* = 0$, $e_s^* \neq 0$. Before yielding, the system does not exhibit torsional motion, all the three elements having the same translational displacement. After yielding of the weakest element (element 3), the resisting force in element 1 remains constant in order to maintain equilibrium ($\Sigma M_{cm} = 0$). The resisting force in element 2 increases with the increase of the load until it yields. When element 2 yields, the system becomes a mechanism. The ultimate value of the load:

$$F_{max} = 2F_{y3} + F_{y2} = F_y\left(1 - \frac{e_s}{d}\right) = F_y\left(1 - \frac{e_s^*}{\left(\frac{3}{2}(\Omega_r^2(1+e_r^{*2}) + e_r^{*2})\right)^{1/2}}\right) = F_y\left(1 - \frac{e_s^*}{\left(\frac{3}{2}\right)^{1/2}\Omega_r}\right) \quad (31)$$

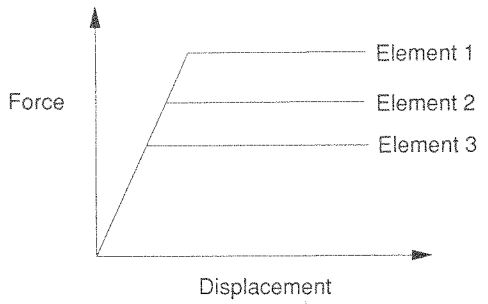

Fig. 18. Element force–displacement relationships for 3-element models with evenly distributed element stiffness.

Equation (31) has been plotted in Fig. 19. The maximum ductility demand of element 3 (the weakest element) is:

$$\mu_3 = 1.0 + \frac{6^{1/2} e_s^*}{\Omega_r - \left(\frac{3}{2}\right)^{1/2} e_s^*} \quad (32)$$

Equation (32) has been plotted in Fig. 20. Comments on the results shown in Figs 19 and 20 are given in Section 4.4.3.

4.4.2.2 Systems with balanced yielding strength

Systems with balanced yielding strength are studied to demonstrate the effect of stiffness eccentricity independent of the effect of strength eccentricity. In this case, the distribution of strength is symmetric, CS coincides with CM but CR does not, that is, $e_s^* = 0$, $e_r^* \neq 0$. All three elements have the same yielding srength but different stiffnesses as shown in Fig. 21. The ultimate load the structure can withstand is $F_{max} = F_y$.

From the equilibrium requirement $\Sigma M_{cm} = 0$, we have $F_1 = F_3$. Because the floor slab is rigid in its own plane, $\Delta_2 = \frac{1}{2}(\Delta_1 + \Delta_3)$, and hence in the elastic range the resisting force in element 2 is:

$$F_2 = k_2\Delta_2 = \frac{1}{3}K_y\frac{1}{2}\left(\frac{F_1}{k_1} + \frac{F_3}{k_3}\right) = F_3\frac{1}{6}K_y\left(\frac{1}{k_1} + \frac{1}{k_3}\right) = F_3\frac{1}{6}K_y\frac{k_1 + k_3}{k_1k_3} \quad (33)$$

Let $k_1 = \alpha K_y$, $0 < \alpha < 1$. From the assumption $k_1 + k_3 = \frac{2}{3}K_y$, we obtain:

$$F_2 = F_3\frac{1}{9\alpha\left(\frac{2}{3} - \alpha\right)} \quad (34)$$

Now, $9\alpha(\frac{2}{3} - \alpha) < 1.0$ if $\alpha \neq \frac{1}{3}$. Furthermore, if $\alpha = \frac{1}{3}$ then $9\alpha(\frac{2}{3} - \alpha) = 1.0$ and in this case $e_r^* = 0$. Therefore, in all cases $F_2 \geq F_3 = F_1$, and element 2 yields first. After the yielding of element 2, the resisting forces in element 1 and 3 remain equal and increase with the increasing load until these two elements yield simultaneously.

The ductility demands of elements *1* and *3* $\mu_1 = \mu_3 = 1.0$ when F reaches F_{max}, whilst the ductility demand of element *2* is:

$$\mu_2 = 1.0 + \frac{3e_r^{*2}}{2\Omega_r^2(1 + e_r^{*2}) - e_r^{*2}} \quad (35)$$

Equation (35) has been plotted in Fig. 22 and the results are discussed in Section 4.4.3.

4.4.2.3 Systems with equal stiffness and strength eccentricities

In this case, the yielding displacements of the resisting elements are equal and the yield strengths of the

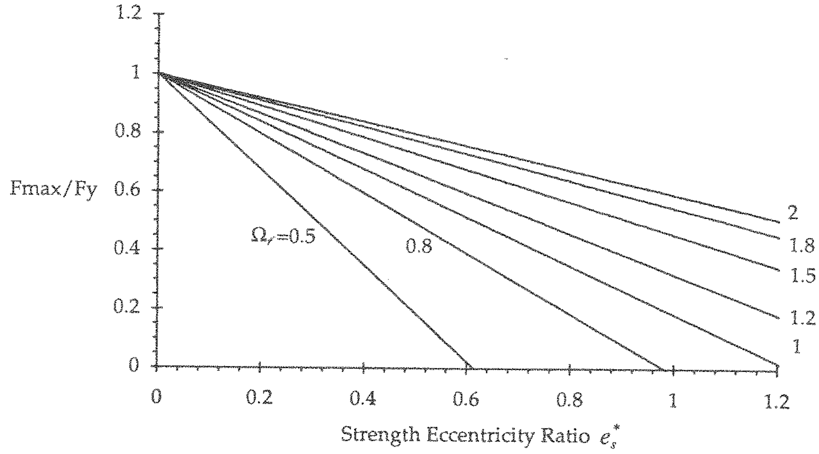

Fig. 19. Effect of strength eccentricity ratio on F_{max} for 3-element models with evenly distributed element stiffness.

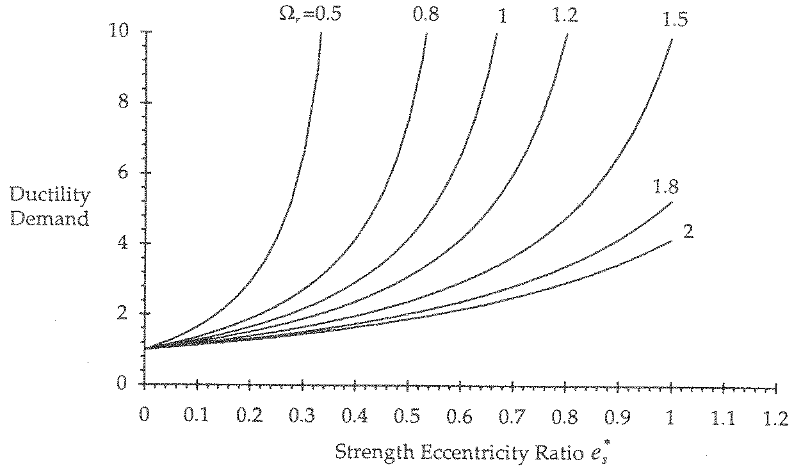

Fig. 20. Effect of strength eccentricity ratio on the ductility demand of element 3 for 3-element models with evenly distributed element stiffness.

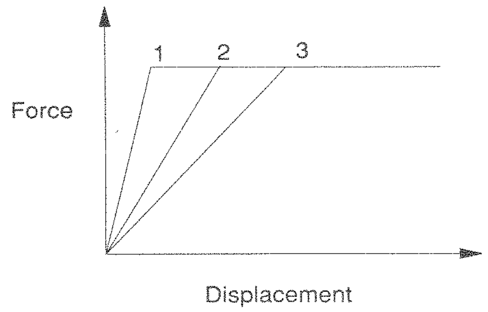

Fig. 21. Element force–displacement relationships for 3-element models with evenly distributed element strength.

elements are proportional to their stiffnesses as shown in Fig. 23.

Because the elements have the same yielding displacement, the one furthest away from CR, element *3*, yields first. Then, the resisting force in element *1* remains constant because of the equilibrium requirement $\Sigma M_{cm} = 0$. Therefore, element *1* becomes stationary and the floor slab rotates about element *1* until element *2* yields. The ultimate value of the applied static load is:

$$F_{\max} = F_y\left(1 - \frac{e_r}{d}\right) = F_y\left(1 - \frac{e_r^*}{\left(\frac{3}{2}(\Omega_r^2(1 + e_r^{*2}) + e_r^{*2})\right)^{1/2}}\right) \tag{36}$$

Equation (36) has been plotted in Fig. 24. The maximum ductility demand occurs in element *3*,

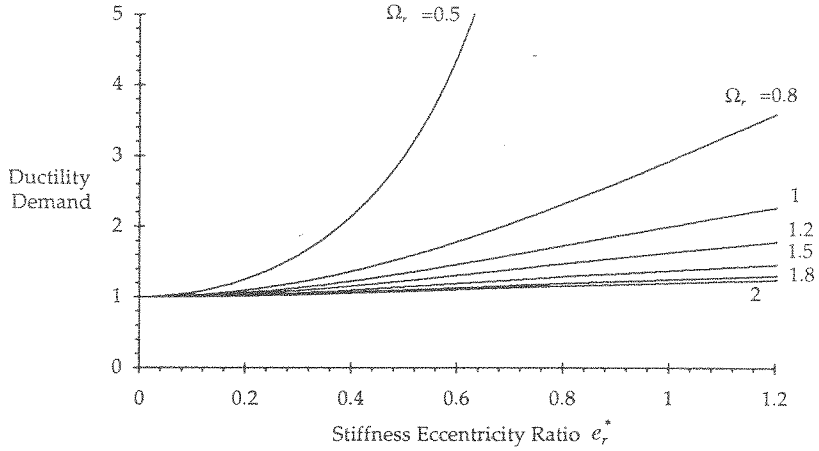

Fig. 22. Effect of stiffness eccentricity ratio on the ductility ratio of element 2 for 3-element models with evenly distributed element strength.

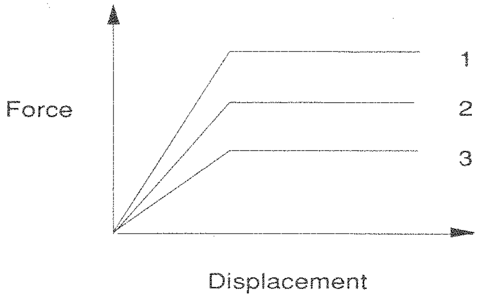

Fig. 23. Element force-displacement relationships for 3-element models with equal strength and stiffness eccentricity.

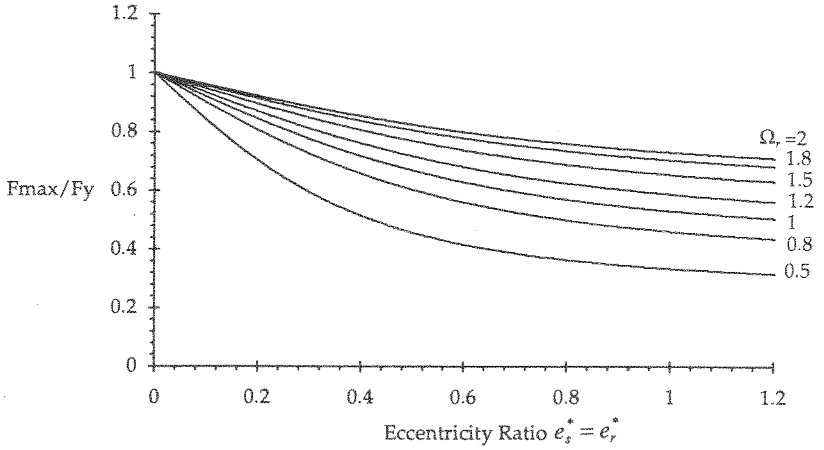

Fig. 24. Effect of strength and stiffness eccentricity ratio on F_{max} for 3-element models with equal strength and stiffness eccentricity.

$$\mu_3 = 1.0 + 6\left\{1 - \frac{e_r^*}{\left(\frac{3}{2}(\Omega_r^2(1+e_r^{*2}) + e_r^{*2})\right)^{1/2}} - \frac{\Omega_r^2\,(1+e_r^{*2})}{\Omega_r^2(1+e_r^{*2}) + e_r^{*2} + e_r^*\left(\frac{3}{2}(\Omega_r^2(1+e_r^{*2}) + e_r^{*2})\right)^{1/2}}\right\} \quad (37)$$

Equation (37) has been plotted in Fig. 25.

4.4.3 Discussions on the results from three element models

Figures 19 and 24 show that the system's strength and stiffness eccentricities, and the uncoupled frequency ratio, are critical parameters affecting the ultimate value of the static load the structure can withstand. Increasing the system's torsional stiffness – i.e. increasing the value of Ω_r – always has the effect of increasing the system's capacity to withstand the static load. The effect of uncoupled frequency ratio on F_{max} is more pronounced for structures with moderate to large eccentricities. F_{max} always declines with increasing values of eccentricity. For torsionally flexible systems ($\Omega_r = 0.5 - 0.8$), a moderate value of eccentricity ratio (0.4) reduces the system's lateral load resisting capacity to about 50–60% of its total yield base shear (Figs 19 and 24). For systems with equal strength and stiffness eccentricities, F_{max} is

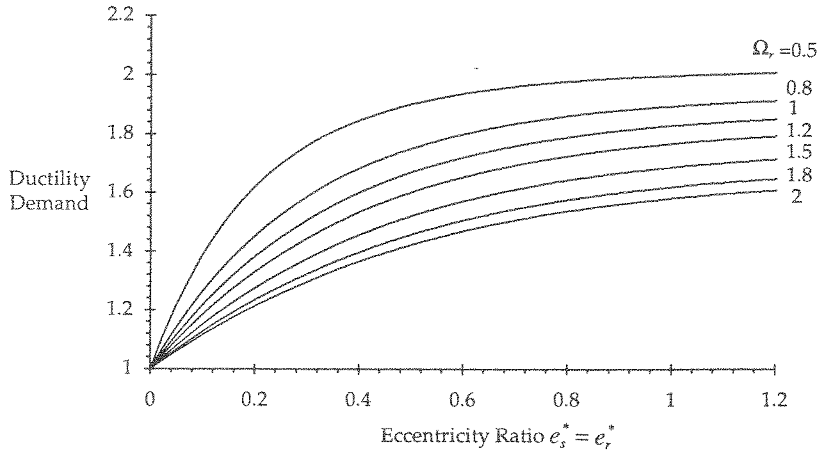

Fig. 25. Effect of strength and stiffness eccentricity ratio on the ductility demand of element 3 for 3-element models with equal strength and stiffness eccentricity.

more sensitive to eccentricity when the uncoupled frequency ratio is low and the eccentricity is small. Comparing Figs 19 and 24 with Figs 16 and 17, it is clear that results obtained from both two-element and three-element models are similar, but the results from three-element models indicate a larger F_{max} compared with two-element models, indicating the effect of development of material plasticity on the structure's capacity to resist lateral loading. Therefore, in static analysis of the effect of inelastic torsional coupling, the two-element model underestimates both the peak ductility demand and the structure's capacity to resist lateral load.

Figure 20 shows the effect of strength eccentricity and uncoupled frequency ratio on the maximum ductility demand of initially symmetric systems. It can be seen that both these system parameters significantly affect the element ductility demand. The maximum ductility demand increases rapidly with increases in strength eccentricity ratio but reduces with increasing uncoupled torsional:translational frequency ratio. The effect of Ω_r is more pronounced for systems with moderate-to-large strength eccentricities than for systems with small strength eccentricities. Increasing the value of Ω_r also has the effect of reducing the influence of strength eccentricity on the maximum ductility demand; the element ductility demand becomes less sensitive to the strength eccentricity for systems with large values of torsional stiffness ($\Omega_r \geqslant 1.8$) than for systems with small-to-moderate values of Ω_r ($\Omega_r = 0.5 - 1.5$).

Figure 22 shows the effect of stiffness eccentricity and uncoupled frequency ratio on the maximum ductility demand of systems with symmetric distribution of yielding strength ($e_s^* = 0$). In this case, the effect of Ω_r is similar to that in the case of initially symmetric systems. The stiffness eccentricity ratio only significantly affects ductility demand of systems with small-to-moderate values of uncoupled torsional:translational frequency ratio ($\Omega_r = 0.5 - 1.0$). The element ductility demand is not sensitive to the stiffness eccentricity ratio when the system's uncoupled frequency ratio is larger than 1.2.

Figure 25 shows the influence of eccentricity and uncoupled frequency ratio on the maximum ductility demand for systems having equal strength and stiffness eccentricity ratios. In this case, element ductility demand is sensitive to both of these parameters. The effect of eccentricity is more pronounced in the range of small-to-moderate eccentricities. In the range of large eccentricities ($e_r^* = e_s^* \geqslant 1.0$), element ductility demand is not sensitive to eccentricity ratio. Larger values of Ω_r always correspond to smaller ductility demand, the effect being more significant for systems with moderate-to-large eccentricity.

5. CONCLUSIONS AND PROPOSALS FOR FURTHER RESEARCH

Torsional coupling of asymmetric building structures to earthquake excitation has been an area of considerable research activity in the past 10 years. However, most existing studies have concentrated on elastic torsional coupling. Research on inelastic torsional coupling is still at an early stage and the results of the few existing studies have led to sharply conflicting conclusions regarding the significance of certain key structural parameters in the inelastic earthquake response. This paper has brought together the results from previous research and carried out a critical assessment in which the model dependency of the dynamic response is highlighted. The ultimate lateral load-carrying capacity and peak element ductility demand in two- and three-element building models have been studied parametrically under static loading conditions. The results have been used both to clarify the differences in the conclusions drawn from previous dynamic response studies and to provide a basis for further research into dynamic torsional response in the inelastic range, with an emphasis on clearer model definition in order to highlight parametric response trends.

It is necessary to carry out further research in the following areas in order to gain a better understanding of the inelastic behaviour of asymmetric building structures under severe earthquake loading and the influence of the various system parameters, and to use the results to provide guidelines for design procedures:

1. Establishment of suitable single and multistorey analytical models which retain simplicity to facilitate the analysis; realism to represent a range of actual building structures; and reliability to ensure conservative estimates of the inelastic torsional coupling effects.

2. Studies of the influence of system parameters on response parameters. The effects of the distribution of mass, stiffness and strength along the horizontal and vertical axes on the inelastic torsional effects are of particular importance.

3. Assessment of current torsional provisions of major building codes in order to incorporate guidelines for design of torsionally asymmetric structures and to ensure realistically conservative estimates of design forces.

4. Studies of the effect of inelastic torsional coupling on the energy absorbing capacities, the collapse and failure processes and the overall stability of asymmetric buildings subjected to severe earthquakes. Hence, to provide guidelines for design procedures to minimize and control the dynamic torsional response, to increase earthquake resistance capacity and to prevent collapse.

A programme of research with the above objectives is currently being undertaken, and includes detailed evaluations of the provisions relevant to torsionally coupled buildings in the Mexican and European earthquake building codes [23–25], together with experimental verification of analytical procedures using small scale model tests carried out on the Science and Engineering Research Council's 6-axis UK national earthquake simulator facility.

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